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Analysis of Solar and Lunar Motions in the Seonmyeong Calendar

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We investigate solar and lunar motions in the Seonmyeong (SM) calendar that was compiled by Xu, Ang of the Tang dynasty (A.D. 618–907) in China and used for 71 years from 822 to 892. This calendar was also used in Korea during the Goryeo dynasty (A.D. 918–1392) and in Japan for 823 years from 862 to 1684, the longest time among the three countries. Referring to historical documents of China, Korea, and Japan, we analyze the calendrical methods of calculating the daily apparent movements of the Sun and Moon in the SM calendar, which were considered their unequal motions, and compare the movements with the results of modern calculations for three periods in the Goryeo dynasty: 919, 1155, and 1392 years (i.e., the beginning, middle, and ending of the dynasty, respectively). We find that a quadratic equation was employed to obtain the daily movement of the Sun using physical quantities on the instant of each solar term, which was tabulated in its calendar book such as the Goryeosa (History of the Goryeo Dynasty). For quantitative analysis, we compute the mean absolute difference (MAD) of the daily apparent movement between the SM calendar and modern calculations and obtain 0.33, 0.30, and 0.31 arcmin for the periods of 919, 1155, and 1392 years, respectively. Meanwhile, we find relatively large MAD values in the daily movement of the Moon: 0.217, 0.284, and 0.240 degrees for each corresponding year. An interesting point is that the MAD value in the lunar motion shows the maximum in 1155 years, and is the minimum in the solar motion. In conclusion, we believe that this study will facilitate in the understanding of the SM calendar further, particularly in the calendrical methods of calculating sunrise, sunset, and eclipse times.

Keywords: historical astronomy, Seonmyeong calendar, ephemeris, Sun, Moon

1. INTRODUCTION

The Xuanming (Manifest Enlightenment) calendar was compiled by Xu, Ang (birth and death years are unknown) of the Tang dynasty (A.D. 618–907) in A.D. 822 and has been used for 71 years in China (Lee 1985). This calendar was also used in Korea and Japan, which was called the Seonmyeong and Senmyo calendars, respectively. According to the preface of the astronomical chapter of the Goryeosa (History of the Goryeo Dynasty), the Goryeo court (A.D. 918–1392) of Korea used the Seonmyeong calendar (hereinafter SM calendar) from the beginning of the dynasty by succeeding it from the Silla kingdom (B.C. 57–A.D. 935) to the end of the dynasty, particularly for calculating solar and lunar eclipses although the Shoushi (Granting the Seasons) calendar of the Yuan dynasty (A.D. 1271–1368) in China was enforced

in 1281 (Seo 2012; Chang 2014; Jun 2014). Therefore, the SM calendar was utilized for 475 years in Korea, at the least. Meanwhile, this calendar was introduced to Japan in A.D. 862 through the Balhae kingdom (A.D. 698–926) of Korea and used for 823 years until 1684 (Kim 2012).

In this study, we analyzed the calendrical methods of solar and lunar motions in the SM calendar referring to the Xintangshu (New History of Tang Dynasty) of China, Goryeosa of Korea, and Senmyo (Senmyo Calendar) and Chokei–Senmyoreki–Sanpo (Calendrical Method for the Senmyo Calendar of the Chokei, shortly Sanpo) of Japan. With regard to Japanese literature, the former is preserved in the National Library of Korea and is known to be made of a Japanese woodblock in 1644 but the author is unknown. Meanwhile, the latter was compiled by Ando, Yueki in 1654 (Soma 2014) and the Chokei (Changqing in Chinese) is the

Chinese reign-title used for four years from 821 to 824.

Together with the Shoushi calendar, the SM calendar is a well-studied Chinese calendar, particularly by Japanese scholars. Sato (1968) reviewed the calendar in the study on the calendar history of Japan. Uchida (1975) utilized the Sanpo for identifying Japanese lunisolar calendar dates with Julian calendar dates. However, Soma (2014) found some mistakes in the daily movement of the Sun, which is tabulated in the Sanpo. According to our examination, the same mistakes exist in the Senmyo preserved in Korea. ISOBDU (1998) compared the contents of the SM calendar recorded on various versions of the Goryeosa with the Sanpo. Qu (2008) studied Chinese calendars including the SM calendar in terms of mathematical techniques (see also Qian 1981; Martzloff 2016). In Korea, the SM calendar was first studied by the scholars of North Korea in 1964 when translating the Goryeosa written in Chinese characters into Korean (EDS 1991). Recently, SADU (2016) retranslated the astronomical chapter of the Goryeosa including detailed annotations and some calculation examples.

The SM calendar is known as a good Chinese calendar, particularly for the calculations of eclipses (Lee 1985). According to the work of Hahn et al. (2016), a quadratic equation of type $x(x\text{-}a)$ was employed in the SM calendar to consider the horizontal parallax of the Moon and was utilized in subsequent Chinese calendars such as the Shoushi calendar. However, the accuracy of the SM calendar has not been analyzed through comparison with modern calculations. The purpose of this study is to investigate the accuracy of the solar and lunar motions by the SM calendar. The remainder of this paper is structured as follows. In Section 2, we introduce the units and astronomical constants in the SM calendar, which are used in this study. We analyze the numerical tables relating to solar and lunar motions listed in calendar books and compare with the results of modern calculations in Section 3. Finally, we summarize our findings in Section 4.

2. UNITS AND ASTRONOMICAL CONSTANTS

In the SM calendar, *Il* (Day), *Bun* (Minute) and *Cho* (Second) were used as the units of an hour (hereinafter, d_H, min_H, and sec_H, respectively) and *Do* (Degree), *Bun* (Arcminute), and *Cho* (Arcsecond) as those of an angle (hereinafter, d_A, min_A, and sec_A, respectively): 1 d_H = 84 or 8400 min_H = 672 or 67200 sec_H (i.e., 1 min_H = 8 sec_H) and 1 d_A = 84 or 8400 min_A = 25200 or 2520000 sec_A (i.e., 1 min_A = 300 sec_A). Because two scales were used in units of min_H and min_A (i.e., 84 and 8400 called the *Gakbeob* and *Seonmyeong-*

Tongbeob, respectively), we distinguished them using the superscripts G (i.e., *Gakbeob*) and T (i.e., *Seonmyeong-Tongbeob*) such as min_H^G or min_A^T.

As in other Chinese calendars, the beginning of the calendrical calculation for a given year, *Y*, was to determine the instant of the winter solstice in its previous year, *Y-1*, called the *Cheonjeong-Dongji* (hereinafter, CJDJ) of year *Y*. In addition, although the calendrical epoch of the SM calendar was the instant of the CJDJ of 822 (i.e., instant of the winter solstice in 821), that of the CJDJ of a given year was obtained by counting from the Super Epoch (i.e., an ancient epoch where the beginnings of the solar, lunar, and sexagenary cycles coincided). The length between the Super Epoch and calendrical epoch of the SM calendar, *Sangwon-Jeoknyeon*, was 7070138 years. The abolition of the Super Epoch is a remarkable characteristic in the Shoushi calendar compared to its previous calendars (Sivin 2008).

Regarding the solar motion, the length of a tropical year called the *Jangse* was 3068055 min_H^T or 365 d_H 2055 min_H^T, and a year was divided into 24 sub-seasons (hereinafter, 24 solar terms) in Chinese calendars. A method of determining the instants of 24 solar terms is to equally divide the length of the tropical year into 24 intervals, as mentioned in Section 3.1. In this case, the length of a solar term is 15 d_H 1835 min_H^T 5 sec_H and this value was called the *Jungjeol*. The celestial sphere was divided into 28 lunar loges (refer to Mihn et al. 2017), and the length of the celestial perimeter called the *Jucheon* is 365 d_A 2153 min_A^T in the Goryeosa. However, the value of the Tang dynasty, i.e., 365 d_A 2153 min_A^T 299 sec_A, is also presented in cases of the *Jucheon* as well as some other constants. When converting the Tang value into units of sec_A, it becomes 920446199 that is recorded as the *Sangsu* in the Goryeosa. Therefore, it appears that the Goryeo astronomer used up to the unit of min_A for the value of the *Jucheon* presumably to simplify the calculations. From the lengths of the tropical year and celestial perimeter, we could deduce that the mean motion of the Sun in the ecliptic was nearly 1 d_A per day although this value was not explicitly defined in historical literature. In addition, the lengths of a synodic month called the *Hapcheok* and of an anomalistic month called the *Yeokjuil* was 29 d_H 4457 min_H^T and 27 d_H 4658 min_H^T 19 sec_H, respectively. Similarly, the 24 solar terms were utilized as the cardinal points in relation to the solar motion, and the periods of 7 d_H 7465 min_H^T and 14 d_H 6529 min_H^T derived from an anomalistic month were utilized as the reference points in relation to the lunar motion as mentioned below. The values of 7465 and 6529 were called the *Chosu* of the 7th and 14th day, respectively, and the values subtracted the *Chosu* from 8400, i.e., 935 and 1871, were called the *Malsu* of the 7th and 14th days, respectively.

Finally, although the mean motion of the Moon among the stars was also not mentioned, it could be deduced as 13 d_A 31 min_A^G per day, as estimated in Section 3.2. In Table 1, we summarize the values of astronomical constants in the SM calendar that are used in this study.

3. SOLAR AND LUNAR MOTIONS

3.1 Solar Motion

The inequality of the solar motion has been known as early as A.D. 570 in China (Needham 1959). In the Goryeosa, the apparent movements of the Sun considering its unequal motion are tabulated with respect to the solar term, as in the case of the Chongxiu-Daming calendar (refer to Choi et al. 2018a). A year was divided into 24 solar terms, as mentioned above, and two methods were utilized for determining the instants of the solar terms: mean and corrected (literal translation) methods. The former method is to equally divide the length of the tropical year into 24 intervals while the latter method is to equally divide the ecliptic into 24 intervals conceptually. Currently, the corrected method is utilized for determining the dates of 24 solar terms (KASI 2018). In both methods, the starting point is the instant of a winter solstice of the preceding year. With regard to calculating the daily movement of the Sun from each solar term, only the calculation procedure is explained in the Goryeosa. In the Japanese literature, meanwhile, the amounts of daily Sun movement in each solar term are tabulated.

We present the ready reckoner relating to the solar motion in Table 2, which is tabulated in the Goryeosa. In column 1 of the table, i represents the solar term notated

Table 2. Ready reckoner in relation to the solar motion

i	ΔU_S^G (min_H^G)	JG (d_H min_H^T sec_H)	U_S^G (min_H^G)	ΔU_S^M (min_H^T)	U_S^M (min_H^T)
1	+60	14 4235 5	0	+449	0
2	+50	14 5235 5	+60	+374	+449
3	+40	14 6235 5	+110	+299	+823
4	+30	14 7235 5	+150	+224	+1122
5	+18	15 0035 5	+180	+135	+1346
6	+6	15 1235 5	+198	+45	+1481
7	−6	15 2435 5	+204	−45	+1526
8	−18	15 3635 5	+198	−135	+1481
9	−30	15 4835 5	+180	−224	+1346
10	−40	15 5835 5	+150	−299	+1122
11	−50	15 6835 5	+110	−374	+823
12	−60	15 7835 5	+60	−449	+449
13	−60	15 7835 5	0	−449	0
14	−50	15 6835 5	−60	−374*	−449
15	−40	15 5835 5	−110	−299	−823
16	−30	15 4835 5	−150	−224	−1122
17	−18	15 3635 5	−180	−135	−1346*
18	−6	15 2435* 5	−198	−45	−1481
19	+6	15 1235 5	−204	+45	−1526
20	+18	15 0035 5	−198	+135	−1481
21	+30	14 7235 5	−180	+224	−1346
22	+40	14 6235 5	−150	+299	−1122
23	+50	14 5235 5	−110	+374	−823
24	+60	14 4235 5	−60	+449	−449

* Corrected values in this study.

with an Arabic number. For reference, i = 1, 7, 13, and 19 represent the winter solstice, spring equinox, summer solstice, and autumnal equinox, respectively. In column 2, ΔU_S^G is the unequal amount of the solar motion in units of min_H^G during the length of each solar term by the mean solar term method (i.e., *Jungjeol*) compared to its mean motion. In column 3, JG is the length of the ith solar term by the corrected solar term method given in units of d_H, min_H^T, and sec_H. Because the Sun moves faster and slower around the winter and summer solstices, respectively, than its mean motion, the values of JG could be obtained by subtracting or adding the value of $\Delta U_S^G \times 100$ to the *Jungjeol* (the units of ΔU_S^G is min_H^G not min_H^T). As an example, the value of JG in the second solar term, JG_2, is 14 d_H 5235 min_H^T 5 sec_H by

Table 1. Summary of astronomical constants in the SM calendar used in this study

Constants	Values	Notes
Gakbeob	84	
Tongbeob	8400	*Seonmyeong-Tongbeob*
Sangwon-Jeoknyeon	7070138 years	Length from the SE[‡] to the epoch of the SM calendar
Jangse	3068055 min_H^T	Tropical year length
Jungjeol	15 d_H 1835 min_H^T 5 sec_H	Solar term length
Jucheon	365 d_A 2153 min_H^T 299 sec_H[†]	Celestial perimeter
Sangsu	920446199 sec_A	Celestial perimeter in units of sec_A
Hapcheok	29 d_H 4457 min_H^T	Synodic month length
Yeokjuil	27 d_H 4658 min_H^T 19 sec_H	Anomalistic month length
7th day Chosu	7465	
7th day Malsu	935	
14th day Chosu	6529	
14th day Malsu	1871	

[†] Value of the Tang dynasty, as indicated in the Goryeosa.
[‡] Super Epoch.

subtracting $+50 \times 100$ min_H^T to the *Jungjeol* (i.e., 15 d_H 1835 min_H^T 5 sec_H). In this sense, the value of 2335 min_H^T for JG_{18} in the Goryeosa is the typographical error of 2435 min_H^T, which is the same that for JG_7. In all tables, the values marked with the asterisk symbol are the corrected values in this study. In column 4, U_S^G is the accumulation of ΔU_S^G until the preceding solar term; hence, the value of U_S^G in the first term, $U_{S,1}^G$, is zero. In column 5, ΔU_S^M is the value dividing $\Delta U_S^G \times 100$ by the mean motion of the Moon per day, $\bar{u}_M^G$. Although the value of $\bar{u}_M^G$ is not defined, it was 13 d_A 31 min_A^G (or 13.3689 d_A), according to our estimation in the next section. Adopting this value, we found that the values of ΔU_S^M were the results rounded half up. In addition, the value of   in the 14th solar term, $\Delta U_{S,14}^M$, should be +374 recorded as the value of the Tang dynasty in the Goryeosa. The last column, U_S^M, is the accumulation of the ΔU_S^M until the preceding solar term; hence, the value of U_S^M in the first term, $U_{S,1}^M$, is zero. The value of $U_{S,17}^M$ is the typographical error in the Goryeosa. Although the Goryeosa states that the value of the Tang dynasty is 1246, it is recorded as 1346 in the version of the Xintangsh that we refer to in this study.

As shown in Table 2, the solar motion is symmetrical around two solstices and equinoxes. The typographical errors discussed above can be verified from this perspective. In reality, however, the Earth revolves around the Sun and its motion is fast around the perihelion and slow around the aphelion. To estimate the accuracy of the solar motion in the SM calendar, we calculated the daily apparent movement of the Sun following the calculation procedures described in the Goryeosa. In Appendix A, we present the procedures using mathematical symbols and formulas to facilitate the understanding of contemporary concepts. As shown in the appendix, a quadratic equation was used to consider the inequality of the Sun, as in the Shoush calendar (refer to Choi et al. 2018b).

3.2 Lunar Motion

The length of the anomalistic month, 27 d_H 4658 min_H^T (19 sec_H), was divided into two ranges referring to the apogee (A) and perigee (P). Hereinafter, Sectors I and II stand for the AP and PA ranges, respectively. To calculate the position of the Moon for a given time, the SM calendar used the daily apparent movement of the Moon in each sector, which was given in a tabular form. In Table 3, we present the table listed in the Goryeosa in relation to the lunar motion. In the table, column 1, denoted as m_A or m_P (integer), represents the day number since the instants of the apogee and perigee passages, respectively (hereinafter, m is the common designation for m_A and m_P). Because the

Table 3. Ready reckoner in relation to the lunar motion

m_A	u_M^G (min$_A^G$)	Δu_M^G (min$_A^G$)	u_M^D (d$_A$ min$_A^G$)	U_M^D (d$_A$ min$_A^G$)	Δu_M^M (min$_H^T$)	U_M^M (min$_H^T$)
1	1012	+14	12 04	0	−830	0
2	1026	+16	12 18	12 04	−726	−830
3	1042	+18	12 34	24 22	−606	−1556
4	1060	+18	12 52	36 56	−471	−2162
5	1078	+18	12 70	49 24	−337	−2633
6	1096	+19*	13 04	62 10	−202	−2970
7	1115	+19	13 23	75 14	−53/+7	−3172
8	1134	+19	13 42	88 37	+82	−3218
9	1153	+19	13 61*	101 79	+224	−3136
10	1172	+19	13 80	115 56	+366	−2912
11	1191	+18	14 15	129 52*	+509	−2546
12	1209	+14	14 33	143 67	+643	−2037
13	1223	+11	14 47	158 16	+748	−1394
14	1234	0	14 58	172 63	+646	−646

m_P	u_M^G (min$_A^G$)	Δu_M^G (min$_A^G$)	u_M^D (d$_A$ min$_A^G$)	U_M^D (d$_A$ min$_A^G$)	Δu_M^M (min)	U_M^M (min$_H^T$)
1	1234	−14	14 58	187 37	+830	0
2	1220	−17	14 44	202 11	+726	+830
3	1203	−18	14 27	216 55	+598	+1556
4	1185	−18	14 09	230 82	+464	+2154
5	1167	−18	13 75	245 07	+329	+2618
6	1149	−18	13 57	258 82	+195	+2947
7	1131	−19	13 39	272 55	+53/−7	+3142
8	1112	−19	13 20	286 10	−82	+3188
9	1093	−19	13 01	299 30	−225	+3106
10	1074	−18	12 66	312 31	−366	+2881
11	1056	−17	12 48*	325 13	−501	+2515
12	1039	−15	12 31	337 61	−628	+2014
13	1024	−12	12 16	350 08	−740	+1386
14	1012	0	12 04	362 24	−646	+646

* Corrected values in this study

day number was counted from the first day (i.e., $m = 1$), the last day number in each sector was the 14th day. In column 2, u_M^G is the apparent movement of the Moon in units of min_A^G on its orbital during the day (i.e., 8400 min_H^T) on the mth day. In column 3, Δu_M^G is the difference of two successive values in u_M^G for m ≤ 13, i.e., $\Delta u_M^G(m) = u_M^G(m+1) - u_M^G(m)$, and 0 for $m = 14$. In column 4, u_M^D is the same quantity with u_M^G but converted in units of d_A and min_A^G, which is listed in a separate table in the Goryeosa. For instance, because the value of u_M^G on the third day in Sector II is 1203 min_A^G ($= 14 \times$ *Gakbeob* + 27 min_A^G), hence that of u_M^D on the day is 14 d_A 27 min_A^G. Suppose that the $\bar{u}_M^G$ is the mean motion of the Moon per day and δu_M^G is the difference between u_M^G and $\bar{u}_M^G$. In columns 6 and 7, Δu_M^M and U_M^G are the physical quantities dividing δu_M^G by $\bar{u}_M^G$ and accumulating Δu_M^M until the preceding day, respectively; hence, U_M^M is zero for $m = 1$. Strictly speaking, the value of Δu_M^M for the mth day, $\Delta u_M^M(m)$, is

$$\Delta u_M^M(m) = \alpha \times \frac{\delta u_M^G(m)}{\bar{u}_M^G(m)/8400} \left[min_H^T \right],$$

(1)

where α is 1 except for $m = 7$ and 14. As shown in Table 3, two values are listed in Δu_M^M for $m = 7$. These values can be obtained from Eq. (1) using the values of the *Chosu* and *Malsu* on the 7th day. That is, two values in each sector can be obtained using α of 7465/8400 and 935/8400. Meanwhile,

the value of Δu_M^M for $m = 14$ is the result using the value of *Chosu* on the 14th day, i.e., α = 6529/8400. However, the value of the daily mean motion of the Moon is not stated in historical literature, as mentioned above. In addition, the daily apparent movement of the Moon in Sector I is not symmetrical to the movement in Sector II, as shown in column 2. Hence, a simple method of estimating the value of $\bar{u}_M^G$ is using the average of the minimum and maximum apparent movements of the Moon, i.e., 1012 and 1234 $\min_A^G$, respectively. Subsequently, we can obtain the $\bar{u}_M^G$ of 1123 $\min_A^G$ (= 13 d_A 31 $\min_A^G$) or 13.1766 degrees. This value is similar to that defined in the Chongxiu-Daming calendar, i.e., 13.1763 degrees (Choi et al. 2018a) or as known in modern astronomy, i.e., 13.176358 degrees (Cox 2002). Applying the $\bar{u}_M^G$ value of 1123 $\min_A^G$ to Eq. (1), we obtained a good agreement with the values of Δu_M^M except for several cases indicating differences of approximately 1 $\min_H^T$. In our opinion, the discrepancies might be caused by the significant figures in the mean motion of the Moon estimated in this study.

3.3 Accuracy Estimation

We estimated the accuracy of the solar and lunar motions by the SM calendar when the calendar was used in Korea by the Goryeo dynasty, and not in China by the Tang dynasty. Hence, we selected the periods of 919, 1155, and 1392 (i.e., the beginning, middle, and ending periods of the Goryeo dynasty, respectively) and compared the amounts of daily Sun and Moon movements listed in the Goryeosa with the results of modern calculations for each period. As mentioned earlier, the first step of the calendrical calculation in the SM calendar is to determine the instant of the CJDJ of a given year. However, in this study, we will not discuss the procedures of calculating the instant of the CJDJ in detail because they are well explained in the work of SADU (2016) where examples are provided. In principle, the instants of the CJDJ and those of the apogee passage are determined by accumulating the lengths of a tropical year and anomalistic month since the Super Epoch. However, it is noteworthy that the day of the winter solstice (also CJDJ) should be placed in the lunar November in Chinese calendars. In

other words, a lunar November is the month containing the winter solstice day. In addition, it is noteworthy that the calendar days are obtained as a sexagenary circle instead of Arabic numbers. Hence, we referred to the work of Han (2001) to convert lunisolar days obtained as a sexagenery circle into Julian calendar days. Because the hour system of the SM calendar was based on the apparent solar time (AST), we also converted the AST at the *Gaeseong* (i.e., a capital of the Goryeo dynasty) into the mean solar time in Korean standard time (KST, i.e., at the meridian of 135° E) by correcting the longitudinal difference, equation of time (ET), and ΔT (i.e., time difference between the terrestrial time and universal time) (see also Lee et al. 2011). We assumed the longitude of the *Gaeseong* as 126° 33′ E, i.e., the longitudinal difference is –33.8 min (Lee 2017), and referred to the work of Morris and Stephenson (2004) for the value of ΔT (e.g., 34.5 min in 918). Finally, we used the astronomical algorithms of Meeus (1998) and the ephemeris of Standish et al. (1997) in modern calculations.

To estimate the accuracy of the solar motion, we first reproduced the instants of the CJDJ of the years used as the examples in the work of SADU (2016) and obtained the same results. Subsequently, we determined the instants of CJDJ of 919, 1155, and 1392 and summarized the results in Table 4 including the perihelion passage times in those days from modern calculations. In the table, YY, MM, DD, and HH represent the year, month, day, and hour, respectively. In the lunisolar date, the day is expressed with the notation utilized by Lee et al. (2012) and the unit of hour is $\min_H^T$ as that in the AST. For instance, #55 November 1154 in the lunisolar calendar corresponds to 15 December 1154 in the Julian calendar, and 1905 $\min_H^T$ in AST at the *Gaeseong* to 4.55 h in KST by correcting the longitudinal difference of –33.8 min, equation of time of –0.72 min, and ΔT of 14.7 min. According to our study, the difference between the CJDJ instant of 1155 and the perihelion passage time at that time was the smallest, i.e., –0.14 days, among the three periods. After determining the CJDJ instant, we computed the dates of the 24 solar terms by accumulating the *Jungjeol* and calculated the daily Sun movement by the SM calendar since the instant of each solar term throughout the procedures described in Appendix A. Subsequently, we compared the

Table 4. Summary of the CJDJ (Cheonjeong-Dongji) instants of 919, 1155, and 1392 by the SM calendar and the perihelion passage times in those days by modern calculations

Instant of the CJDJ (S)								Perihelion Passage Time (M)				S–M	ΔT	ET
Lunisolar Date [AST]				Julian Date [KST]				Julian Date [KST]						
YY	MM	DD	HH	YY	MM	DD	HH	YY	MM	DD	HH	DD	min	min
918	11	#17	4125	918	12	16	12.37	918	12	13	3.59	3.37	34.5	–0.40
1154	11	#55	1905	1154	12	15	4.55	1154	12	15	7.79	–0.14	14.7	–0.72
1391	11	#38	1740	1391	12	14	4.53	1391	12	15	15.82	–1.47	5.5	–0.89

movements for 15 days in each solar term (i.e., for a total of 360 days) with the results of modern calculations. Hence, the interval between the last day in a solar term and the first day in the next solar term (i.e., next solar term day itself) is 1 d_H 1835 min_H^T 5 sec_H, and not 8400 min_H^T (i.e., a day).

Fig. 1 shows the differences in the daily apparent movement of the Sun between the SM calendar, S^{SM}, and modern calculations, S^{MC}, for the periods of 919, 1155, and 1392. The horizontal axis is the elapsed date since the CJDJ instant of each year and the vertical axis is $S_i^{SM}(n) - S_i^{MC}(n)$ in units of (modern) arcmin, where i is the solar term from 1 to 24 and n is the day number since each solar term from 0 to 14. In the figure, the red solid, green dot, and blue short-dash lines represent the results for the periods of 919, 1155, and 1392, respectively; the vertical dotted lines represent the elapsed dates of the spring equinox, summer solstice, and autumn equinox, from left to right, since the CJDJ instant of each year. For a quantitative comparison, we also estimated the mean absolute difference (MAD) value defined as follows:

$$\text{MAD} = \frac{1}{N} \sum_{i=1}^{i=N} \left| X^{SM} - X^{MC} \right|_i , \qquad (2)$$

where X^{SM} and X^{MC} are physical quantities from the SM calendar and modern calculations, respectively, and N is the number of data used in the calculations. We found that the MAD values are 0.33, 0.30, and 0.31 arcmin for the periods of 919, 1155, and 1392, respectively. In addition, the pattern in Fig. 1 is similar to that in the result of the Chongxiu-Daming calendar of the Jin dynasty for the solar motion: the maximum differences are indicated around the winter and summer solstices, and the minimum around the spring and autumn equinoxes (refer to Choi et al. 2018a).

In the lunar motion section of the Goryeosa, the methods of determining the elapsed dates of four phases of the Moon (i.e., new, first quarter, full, and last quarter moons) since the instant of an apogee passage are described. In addition, the instant of the apogee passage to calculate the elapsed date of the new moon in lunar November was computed using the length of the *Yeonyeo* (i.e., the difference in instants between the CJDJ and apogee passage around that time). For this study, however, we obtained the instant of the apogee passage immediately before the CJDJ instant (hereinafter, the CJAP instant) by accumulating the length of the anomalistic month from the Super Epoch. In Table 5,
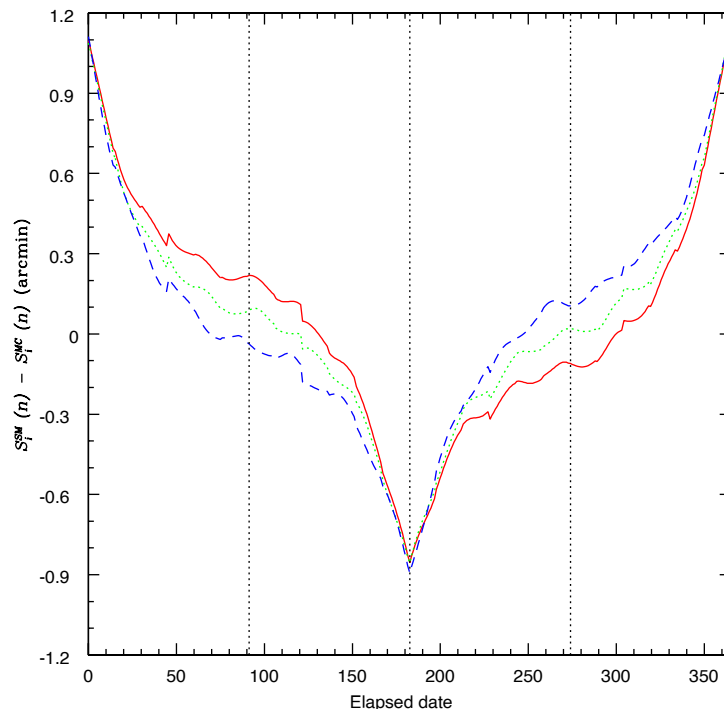

Fig. 1. Differences in daily solar motion between the SM calendar, S^{SM}, and modern calculations S^{MC}: red solid, green dot, and blue short-dash lines represent the results for the periods of 919, 1155, and 1392, respectively. The horizontal axis is the elapsed dates since the CJDJ (Cheonjeong-Dongji) instant of each year and the vertical axis is $S_i^{SM}(n) - S_i^{MC}(n)$ in units of (modern) arcmin. The vertical dotted lines are the elapsed dates of the spring equinox, summer solstice, and autumn equinox, from left to right, since the CJDJ instant of each year.
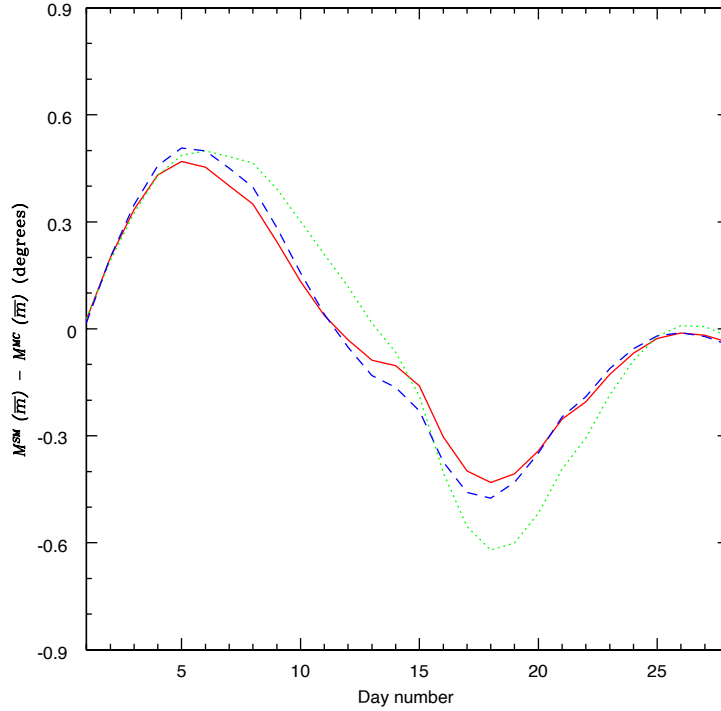
Fig. 2. Differences in daily lunar motion between the SM calendar, M^{SM}, and modern calculations, M^{MC}: red solid, green dot, and blue short-dash lines represent the results for the periods of 919, 1155, and 1392, respectively. The horizontal axis is the day number since the instant of the apogee passage and the vertical axis is $M^{SM}(\bar{m}) - M^{MC}(\bar{m})$ in units of (modern) degrees, which are averaged over the 12 cycles of anomalistic month length for each day number.

we present the instants of the CJAP referring to the instants of the CJDJ of 919, 1155, and 1392 together with the results of modern calculations.

To estimate the accuracy of the lunar motion in approximately one tropical year, we first determined 12 successive instants of the apogee passage in each year by accumulating the anomalistic month length to the CJAP instant. Subsequently, we calculated the daily Moon movement during each anomalistic month length using the DE405 ephemeris. As indicated in Section 3.2, however, the lunar movement listed in Table 3, u_M^G, is the daily amount (i.e., for 8400 min_H^T) for 28 days since the instant of an apogee passage although the length of the anomalistic month is 27 d_H 4658 min_H^T 19 sec_H. Therefore, in modern calculations for the daily lunar movement, we determined the beginning point of each day by accumulating 8400 min_H^T from the

instant of an apogee passage but reset the point on the days of $m_A = 8$ and $m_P = 1$ and 8 using the *Chosu* of the 7th and 14th days. For example, the beginning point on $m_A = 8$ is 7 d_H 7465 min_H^T and not 8 d_H 0 min_H^T if the instant of each apogee passage is 1 d_H 0 min_H^T. From the comparison with the movements listed in the Goryeosa, we found that the MAD values are 0.217, 0.284, and 0.240 (modern) degrees for the periods of 919, 1155, and 1392, respectively. In Fig. 2, we present the differences in daily Moon movement between the SM calendar, M^{SM}, and modern calculations, M^{MC}. The horizontal axis is the day number since the instant of an apogee passage, and the vertical axis is $M^{SM}(\bar{m}) - M^{MC}(\bar{m})$ in units of (modern) degrees, which are averaged over the 12 cycles of anomalistic month length for each day number. In the figure, the legend is the same as that in Fig. 1: red solid, green dot, and blue short-dash lines represent the results for

Table 5. Summary of the instants of CJAP (apogee passage immediately before the CJDJ) of 919, 1155, and 1392 by the SM calendar and the differences with apogee passage times in those days by modern calculations

Instants of the CJAP (S)								Apogee Passage Times (M)				S–M	ΔT	ET
Lunisolar Calendar [AST]				Julian Calendar [KST]				Julian Calendar [KST]						
YY	MM	DD	HH	YY	MM	DD	HH	YY	MM	DD	HH	DD	min	min
918	11	#01	3684	918	11	30	10.95	918	12	1	5.06	–0.75	34.5	7.37
1154	11	#32	0503	1154	11	22	1.87	1154	11	23	9.85	–1.33	14.7	10.54
1391	11	#28	3736	1391	12	4	11.10	1391	12	5	11.21	–1.00	5.5	5.10

the daily Moon movement for the periods of 919, 1155, and 1392, respectively.

4. SUMMARY

We investigated the solar and lunar motions in the SM calendar of the Tang dynasty in China. According to the preface of the Goryeosa, this calendar was used in Korea during the Goryeo dynasty succeeding the Silla Kingdom. Although it was known that the Goryeo dynasty introduced the Shoushi calendar in 1281, the SM calendar had been utilized until the end of the dynasty for the prediction of solar and lunar eclipse times. Primarily referring to the Goryeosa, we analyzed the calendrical methods of the SM calendar to calculate the solar and lunar motions. Subsequently, we compared the motions with modern calculations using the DE405 ephemeris for three periods: 919, 1155, and 1392, i.e., the beginning, middle, and ending of the Goryeosa dynasty, respectively. Regarding the solar motion, we found that a quadratic equation was employed to calculate the daily Sun movement. Furthermore, we found that the MAD values were approximately 0.31 arcmin for the three periods, although the minimum was indicated in 1155, where the CJDJ instant of 1155 was the closest to the perihelion passage time among the three periods. Meanwhile, we found that relatively large MAD values for the lunar motion: 0.247 degrees on average. In addition, we found that the MAD value indicated the maximum in 1155, unlike the cases of the solar motion. This might be the reason for the largest difference in apogee passage time between the SM calendar and modern calculations in 1155 among the three periods analyzed in this study.

In conclusion, we believe that this study will facilitate in the understanding of calendrical methods in the SM calendar, such as the sunrise and sunset times, eclipse times, and so forth.

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APPENDIX A. Calculation Methods of The Daily Sun Movement

According to the work of Qu (2008), the method of calculating the daily Sun movement was based on the Dayan calendar of the Tang dynasty. In the following, we formulated the calculation procedures explained in the Goryeosa. First, calculate the physical quantities of the *Jungryul (JR), Hapcha (HC), and Jungcha (JC)*:

$$JR_i = \Delta U^G_{S,i} \times \alpha \, / \, JG_i \qquad (A1)$$

$$HC_i = JR_i - JR_{i+1} \qquad (A2)$$

$$JC_i = HC_i \times JG_i \, / \, (JG_i + JG_{i+1}) \, , \qquad (A3)$$

where i represents the ith solar term; $\Delta U^G_{S,i}$ and JG_i are physical quantities in the ith solar term, which are given in Table 2, and α is the *Gakbeob* (i.e., 84). In addition, the value of JR_{i+1} in Eq. (A.2) should be substituted as JR_{i-1} in the calculations of HC_i for i = 6, 8, 12, and 18 (i.e., solar terms incurring the sign change in the value of ΔU^G_S compared to each subsequent solar term). Subsequently, compute the following quantities: *Ilcha* (IC), *Choryul* (CR), *Malryul* (MR), *Chojeongryul* (CJR), and *Maljeongryul (MJR)*.

$$IC_i = JC_i \times 2 \times \beta \, / \, JG_i \qquad (A4)$$

$$CR_i = JC_i + JR_i \, , \quad MR_i = JC_i - JR_i \qquad (A5)$$

$$CJR_i = CR_i - IC_i \, / \, 2 \, , \quad MJR_i = MR_i + IC_i \, / \, 2 \, , \qquad (A6)$$

where β is 100. In the SM calendar, the addition or subtraction operation differs with respect to the solar term in calculating the values of CR_i, MR_i, CJR_i, and MJR_i. However, these values can be obtained from Eqs. (A.5) and (A.6) if adopting the signing system given in Table 2. Finally, the dai-

ly inequality of the Sun (or equation of center of the Sun) in the ith solar term is obtained by cumulatively subtracting IC_i from CJR_i. That is, the unequal amount of the solar motion on the nth day in the ith solar term, $\Delta U^G_{S,i}(n)$, is

$$\Delta U^G_{S,i}(n) = CJR_i - \sum_{j=0}^{j=n} IC_i \text{ for } 0 \le n \le 14$$

$$\Delta U^G_{S,i}(n) = MJR_i \times \gamma \text{ for } n = 15 \, , \qquad (A7)$$

where n is the day number (integer) from 0 (i.e., day of each solar term) to 15, and γ is the decimal fraction when converting the length of the *Jungjeol* into units of d_H (i.e., $\frac{1835+5/8}{8400}$).

According to our examination, however, the values of α and β should be a unity (see also Martzloff 2016). This can be verified by summing $\Delta U^G_{S,i}(n)$ for n from 0 to 15. That is, the value of $\sum_{n=0}^{n=15} \Delta U^G_{S,i}(n)$ for a given solar term should equal to that of $\Delta U^G_{S,i}$. In addition, we reproduced the amounts of ΔU^M_S on the nth day in the ith solar term, $\Delta U^M_{S,i}(n)$, using the same method to obtain $\Delta U^G_{S,i}(n)$. Except for $\Delta U^M_{S,2}(n)$, in which their errors were already discussed by Soma (2014), we found an exact agreement with the values listed in the Senmyo, a Japanese SM calendar preserved in the National Library of Korea, and the Sanpo. Finally, the unequal amount of solar motion until the nth day in the ith solar term, $U^G_{S,i}(n)$, is calculated as follows:

$$U^G_{S,i}(n) = U^G_{S,i} + \sum_{j=1}^{j=n} \Delta U^G_{S,i}(n) = U^G_{S,i} + \sum_{j=1}^{j=n} (CJR_i - \sum_{k=0}^{k=j} IC_i) \, . \quad (A8)$$

It can be rewritten as

$$U^G_{S,i}(n) = U^G_{S,i} + n \times CJR_i - \frac{n(n-1)}{2} \times IC_i \, . \qquad (A9)$$

This equation indicates that a quadratic equation was utilized to consider the inequality of the Sun.